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Calculation of Wing-Body Pressures in Incompressible Flow Using Green's Function Method

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In the present paper, the finite element technique is applied to the calculation of three-dimensional steady incompressible potential flow around a wing-body combination, the main emphasis being placed upon the study of the aerodynamic interference effect between the wing and the body. The formulation of the problem is made in the form of an integral equation following Morino's method. By the use of the finite element technique combined with the collocation method, this integral equation is reduced to a set of linear algebraic equations. By solving this set, the pressure distribution over the surface of the wing-body combination is obtained. Wind tunnel experiments have been conducted at National Aerospace Laboratory of Japan in pace with the numerical analysis. It has been found that agreement between the numerical results obtained by the present method and those obtained by the wind tunnel experiments is very encouraging.

Introduction

An important and challenging aerodynamic problem for the aerospace engineer is the evaluation of the pressure distribution on the surface of a flight vehicle, not only from the viewpoint of performance, but also of aeroelasticity, stability, and control. Extensive investigations have been and are being conducted involving numerical analysis of the problem as described in Refs. 1-9, for example. However, the flight vehicle has a complicated configuration in general, and numerical analysis requires tremendous computation labor and time. Consequently there has been a strong demand for the aerospace engineer, who is currently struggling with the rapid development of computer-aided design (CAD), to establish a computation procedure which can solve this kind of problem numerically with good accuracy and in reasonable computing time.

In the present paper, we consider the flow around a streamlined wing-body combination, which frequently will be called an airplane hereafter for the sake of simplicity, and put main emphasis on the aerodynamic interference effect between wing and body. We confine our interest to steady and incompressible flow, and assume the wing-body combination to be stationary. We also assume the flow to be inviscid and irrotational, remembering that the flow around a streamlined flight vehicle can be assumed to be potential for many practical cases.

In the present paper, Morino's method[10-12] is used for formulating the problem in the form of an integral equation. By the use of the finite element technique combined with the collocation method, which is sometimes called a panel method, this integral equation is reduced to a set of linear algebraic equations. By solving the set, we obtain values of the disturbance velocity potential on each element, which are then used for numerically calculating the pressure distribution over the surface of the wing-body combination.

The present paper has three distinguishing characteristics:

1) A method is proposed for numerical calculation of the pressure acting over the airplane surface, by the use of values of the velocity potential Φ at the centroids of four adjacent quadrilateral elements, as explained later in the section entitled "Discretization."

2) Wind tunnel experiments have been conducted along with the authors' numerical analysis, by T. Ichikawa and M. Yanagizawa and their laboratory staff at the National Aerospace Laboratory of Japan for checking the validity of the present numerical method. It has been found that agreement between the numerical results obtained by the present method and those by the wind tunnel experiments is very encouraging.

3) The computing technique proposed here has been found to give desired results with good accuracy and with reasonable computing time, and to suggest that it can be used for practical design purposes.

Formulation of the Problem

We consider a wing-body combination placed in a uniform flow U_∞ as shown in Fig. 1. Assuming the flow around the wing-body combination to be steady, incompressible, inviscid, and irrotational, we have the governing equation for the flow as

$$\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial y^2}+\frac{\partial^2\varphi}{\partial z^2}=0 \tag{1}$$

while the boundary condition is given by

$$\frac{\partial\varphi}{\partial n}=-U_\infty\cdot n \qquad \text{on } S_A \tag{2}$$

where φ is the disturbance velocity potential which is related to the velocity potential $\Phi(x,y,z)$ and the velocity $v(x,y,z)$ of the flow by the following relations:

$$v=\text{grad}\Phi=U_\infty+\text{grad}\varphi \tag{3}$$

In Eq. (2), S_A is the surface of the airplane, and n is unit normal drawn on S_A into the interior of the flow. The

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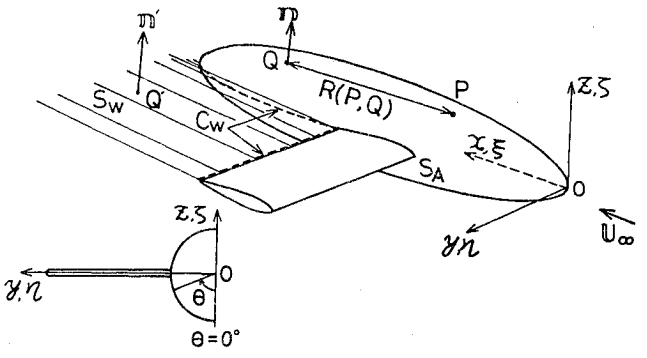

Fig. 1 Coordinate system.

pressure, denoted by p, is derivable from Φ and is given in a nondimensional form as follows:

$$C_p = \frac{p - p_\infty}{(1/2)\rho U_\infty^2} = 1 - (\text{grad}\Phi)^2 / U_\infty^2 \tag{4}$$

where p_∞ is the pressure of the uniform flow, U_∞ is the magnitude of U_∞, and ρ is the density of the flow.

Now, some consideration will be given to the wake. We assume the wake surface, denoted by S_w, to be a sheet of zero thickness and denote the intersection curve between S_A and S_w by C_w. It is obvious that the location of C_w is determined by the effect of viscosity existing in the flow. Namely, if it were possible to formulate and solve governing equations for the flow, in which the effect of viscosity has been taken into account, the location of C_w could be determined automatically. Since we are assuming the flow to be inviscid for the sake of simplicity, however, the location of C_w should be assumed.

It is well-established, following the Kutta condition, that we may take C_w to coincide with the trailing edge of the wing as far as the rear part of the wing is concerned. For the rear part of the body, however, no definite hypothesis seems ever to have been given concerning the location of C_w. Consequently, we shall assume C_w to be located at the rear part of the body, taking experimental evidence into consideration.

The wake surface S_w is assumed to consist of a family of streamlines issuing from C_w. Since the pressure is continuous across C_w, we conclude that the difference of φ across the wake, denoted by $\Delta\varphi$, is constant along an arbitrary streamline in the wake and may be taken as

$$\Delta\varphi = \varphi_+ - \varphi_- \tag{5}$$

where φ_+ and φ_- are values of φ on S_A at two points located just above and below a point belonging to C_w, respectively, from which the streamline under consideration is shed out.

With these preliminaries, and by the use of Morino's method,[10-12] the present problem is formulated in the form of an integral equation as follows:

$$2\pi\varphi(P) - \iint_{S_A} \varphi(Q) \frac{\partial}{\partial n(Q)} \left[\frac{1}{R(P,Q)}\right] dS - \iint_{S_w} \Delta\varphi(Q') \frac{\partial}{\partial n'(Q')} \left[\frac{1}{R(P,Q')}\right] dS = \iint_{S_A} U_\infty \cdot n(Q) \frac{1}{R(P,Q)}\, dS \tag{6}$$

where P and Q are two arbitrary points on S_A, Q' is an arbitrary point on S_W, while $R(P,Q)$ and $R(P,Q')$ are the distances between P and Q, and P and Q', respectively. n' is unit normal drawn on the upper surface of the wake S_W. $\iint$ means that Cauchy's principal value should be taken. It is seen that the only unknown function in Eq. (6) is φ on S_A.

Discretization

The integral Eq. (6) is discretized by the following procedures (see Fig. 2):

1) S_A is discretized into quadrilateral elements Σ_k; $k = 1,2,\ldots,N$. Values of φ and $U_\infty \cdot n$ in each element are represented by those at its centroid and assumed constant in each element.

2) S_W is discretized into semi-infinite strip elements $\Sigma_{k'}$. Values of $\Delta\varphi$ are assumed constant in each strip element.

3) Values of φ_+ and φ_- are approximated by those of φ belonging to two elements on S_A, which have, as a common side, the same part of C_W from which the strip element under consideration issues. These two elements will be called the upper or lower element, depending upon whether the element is located above or below the C_W curve.

By the use of the discretization mentioned above, the unknown function φ over S_A has been approximated by N degrees of freedom. By choosing the centroid, denoted by P_h, of the hth element Σ_h as the control point, we have from Eq. (6) the following algebraic equations:

$$[\delta_{hk} - C_{hk} - W_{hk}]\{\varphi_k\} = [b_{hk}]\{-U_\infty \cdot n_k\} \tag{7}$$

where δ_{hk} is the Kronecker symbol, while C_{hk}, W_{hk}, and b_{hk} are aerodynamic influence coefficients defined by

$$C_{hk} = \left[\frac{1}{2\pi} \iint_{\Sigma_k} \frac{\partial}{\partial n}\left(\frac{1}{R}\right) dS\right]_{P=P_h} \tag{8a}$$

$$b_{hk} = \left[\frac{1}{2\pi} \iint_{\Sigma_k} \frac{1}{R}\, dS\right]_{P=P_h} \tag{8b}$$

$$W_{hk} = \left[\pm\frac{1}{2\pi} \iint_{\Sigma_{k'}} \frac{\partial}{\partial n}\left(\frac{1}{R}\right) dS\right]_{P=P_h} \tag{8c}$$

It is noted here that for an element which does not contain a part of C_w in its boundary, we take $W_{hk} = 0$, and that for an element which contains a part of C_W in its boundary, we have $W_{hk} \neq 0$, where the plus (or minus) sign in the right-hand side of Eq. (8c) is taken if the element is an upper (or lower) element. It is also noted here that since each of the quadrilateral elements is not in one plane in general, the integrations of Eqs. (8) are conducted by projecting the quadrilateral element into a plane, as explained in Ref. 12.

For obtaining the velocity and pressure distribution over S_A, we sum up the potential of the uniform velocity U_∞ and the value of the disturbance velocity potential φ_k to obtain the velocity potential Φ in each element. Next, we denote the centroids of four adjacent elements by P_i; $i = 1,2,3,4$ as shown in Fig. 3, and values of the velocity potentials at these points by Φ_i; $i = 1,2,3,4$, respectively. Furthermore, we denote the middle points of the four sides of the quadrilateral $P_1P_2P_3P_4$ by R_1, R_2, R_3 and R_4, and the unit vectors in the directions of $\overrightarrow{R_3R_1}$ and $\overrightarrow{R_2R_4}$ by λ_1 and λ_2, respectively. With these preliminaries, derivatives of Φ in the directions of λ_1 and λ_2 are calculated as follows:

$$\frac{d\Phi}{d\lambda_1} = \frac{\Phi_1 + \Phi_2 - \Phi_3 - \Phi_4}{|P_1 + P_2 - P_3 - P_4|} \tag{9a}$$

$$\frac{d\Phi}{d\lambda_2} = \frac{\Phi_1 - \Phi_2 - \Phi_3 + \Phi_4}{|P_1 - P_2 - P_3 + P_4|} \tag{9b}$$

where P_1, P_2, P_3, and P_4 are position vectors of the points P_1, P_2, P_3, and P_4, respectively.

Next, we take the a_1 axis in the direction of λ_1, and the a_2 axis in the direction perpendicular to λ_1 in the plane spanned by λ_1 and λ_2. Denoting the unit vectors in the directions of the

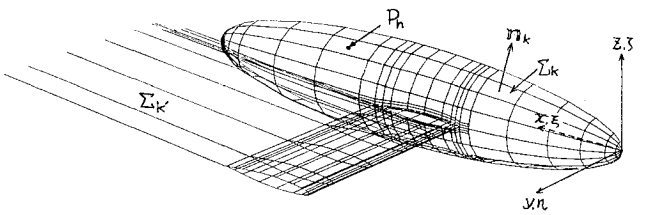

Fig. 2 Mesh division of mid-wing airplane.

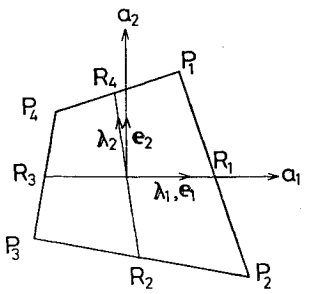

Fig. 3 Local coordinate used in calculation of C_p.

a_1 and a_2 axes by e_1 and e_2, respectively, we have

$$\Phi_{a_1} = \frac{\partial \Phi}{\partial a_1} = \frac{d\Phi}{d\lambda_1} \tag{10a}$$

$$\Phi_{a_2} = \frac{\partial \Phi}{\partial a_2} = \left\{ \frac{d\Phi}{d\lambda_2} - (\lambda_2 \cdot e_1) \frac{d\Phi}{d\lambda_1} \right\} \Big/ (\lambda_2 \cdot e_2) \tag{10b}$$

By the use of these relations, we obtain the velocity and the nondimensionalized pressure at the intersection point of the a_1 and a_2 axes as follows

$$(\Phi_{a_1}^2 + \Phi_{a_2}^2)^{1/2} \tag{11}$$

and

$$C_p = 1 - (\Phi_{a_1}^2 + \Phi_{a_2}^2)/U_\infty^2 \tag{12}$$

As mentioned above, we have assumed a constant singularity distribution in each element. It is noted here that the constant singularity formulation is accurate only when the surface velocities are computed by differentiating the velocity potential given at the element control points. The off-body velocities near the configuration are not accurate since they are computed by summation of velocity influence coefficients and this limits the general applicability of the approach based upon the constant singularity distribution.

Mid-Wing Airplane

First, the mid-wing airplane shown in Fig. 1 is treated. We choose the shape of the body to be an ellipsoid of revolution. We take the x axis in coincidence with the axis of revolution of the body, the y axis in the starboard direction, and the z axis in the upward direction, while the origin of the coordinate system is taken at the nose of the body. For defining the coordinates of the surface of the body, two parameters x and θ are employed, where θ is the angle measured from the bottom of the body as shown in Fig. 1. A curve on the surface of the body, along which θ is constant, will be called a meridian.

The wing is assumed to have a rectangular planform, symmetrical airfoil section, no twist, and no dihedral. It is attached to the body with zero incidence angle at ± 90 deg meridian positions as shown in Fig. 1. The flow velocity U_∞ is assumed to be in the (x,z) plane and the angle of attack α is

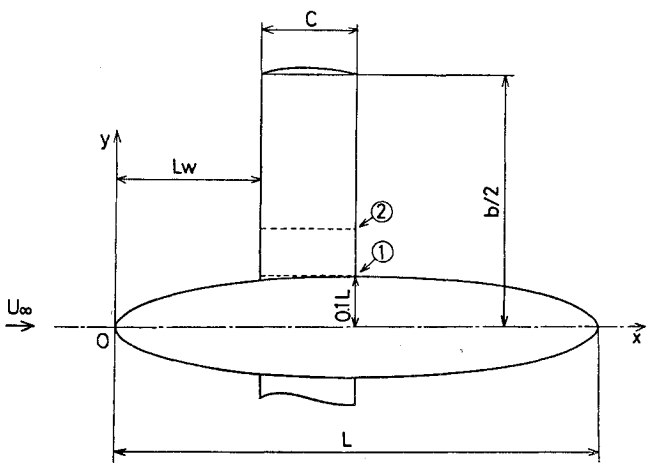

Fig. 4 Plan view of mid-wing airplane.

assumed small. Since the symmetry of the flowfield has been thus assured, only half of the flowfield is to be analyzed.

For the analysis of the mid-wing airplane, we shall employ the following two assumptions:

1) On the location of C_w on the rear part of the body: The curve C_w on the rear part of the body is assumed to be in coincidence with ± 90 deg meridian curves and continuous with the trailing edges of the wing.

2) On the shape of S_w: The wake surface S_w is assumed to consist of a family of semi-infinite straight lines which start from C_w and extend to infinity in the direction parallel to the uniform flow U_∞.

Obviously, the flow separation occurs somewhere in the tail part of the body and the spread of the separated region depends upon the angle of attack. However, we employed the first assumption taking experimental evidence, which will be explained later, into account and remembering that the wake surface S_w has been assumed to be a sheet of zero thickness. The second assumption may be removed if the shape of S_w is determined by an iterative method such as that mentioned in Ref. 13, for example. However, the second assumption is employed in our formulation because of its simplicity, and also because the use of the second assumption will probably not deteriorate the accuracy of the numerical results significantly as far as the pressure distribution over S_A is concerned.

By the use of the above method, the pressure distribution has been calculated for a mid-wing airplane, of which the plan view is shown in Fig. 4. The wing is of a rectangular planform having aspect ratio of 5, namely $b/c = 5$, where b and c are the span and the chord of the wing, respectively, while the body is an ellipsoid of revolution having fineness ratio of 5. The span of the wing is taken equal to the length of the fuselage denoted by L, namely, $b = L$. The wing has NACA 65A010 section and is fixed to the body with zero incidence angle at $L_w = 0.3\ L$ and $\theta = 90$ deg, where L_w is the position of the leading edge of the wing. Two spanwise stations, $y/c = 0.513$ and 1.025, denoted by ① and ②, respectively, are shown in the same figure for later reference.

Mesh division is made as shown in Fig. 2. Due to the symmetry property of the present problem, only the starboard half was treated, dividing the wing surface into 126 elements and the body surface into 238 elements, while S_w in the rear parts of the wing and the body is divided into 7 and 8 strip elements, respectively.

Figure 5 shows pressure distributions over the wing surface at two spanwise stations, $y/c = 0.513$ and 1.025 for $\alpha = 3.74$ deg. In the figure, the abscissa is the nondimensionalized chordwise coordinate x/c, where $x/c = 0$ and 1.0 correspond to the leading and trailing edges, respectively, while the ordinate is the nondimensionalized pressure C_p defined by Eq.

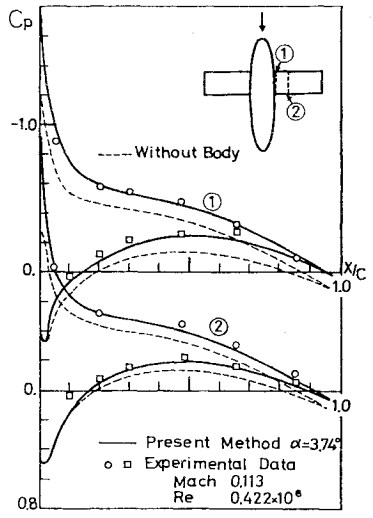

Fig. 5 Chordwise pressure distributions acting on wing surface at $\alpha = 3.74$ deg.

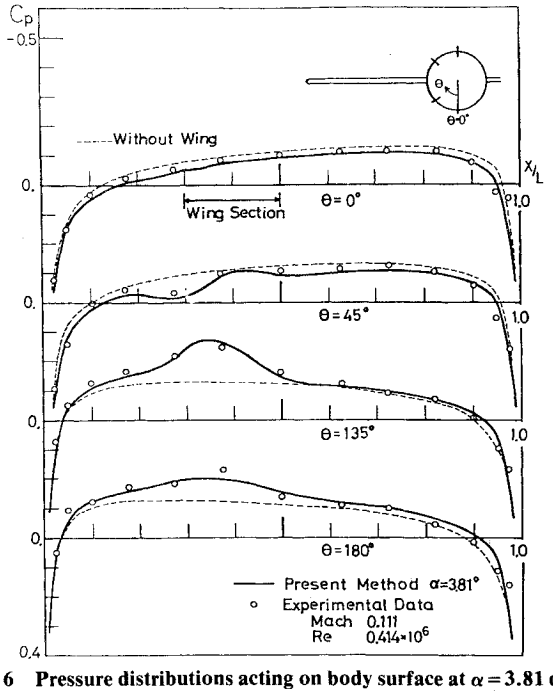

Fig. 6 Pressure distributions acting on body surface at $\alpha = 3.81$ deg.

(4). Figure 6 shows pressure distributions over the body surface along four meridian curves: $\theta = 0$, 45, 135, and 180 deg for $\alpha = 3.81$ deg. In the figure, the abscissa is nondimensionalized longitudinal coordinate x/L, where $x/L = 0$ and 1.0 correspond to the nose and tail of the body, respectively, while the ordinate is C_p. "Wing section" in the figure indicates the position of the wing. In these figures, the solid lines represent results obtained by the present method, while discrete symbols, o and □, represent experimental results obtained by the National Aerospace Laboratory of Japan.[14] It is seen that agreement between the numerical and experimental results is very good. It is noted here that the dotted lines in Fig. 5 indicate the pressure distribution over the wing without the body, namely the wing alone, while those in Fig. 6 indicate the pressure distribution over the body without the wing, namely the body alone. The effect of interference between the wing and body can be clearly observed from these figures.

The flow velocity vectors over the body surface were also calculated for $\alpha = 3.81$ deg. On the other hand, a wind tunnel

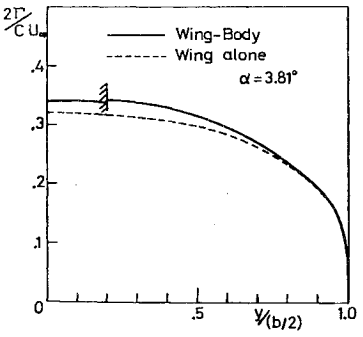

Fig. 7 Spanwise circulation distributions at $\alpha = 3.81$ deg.

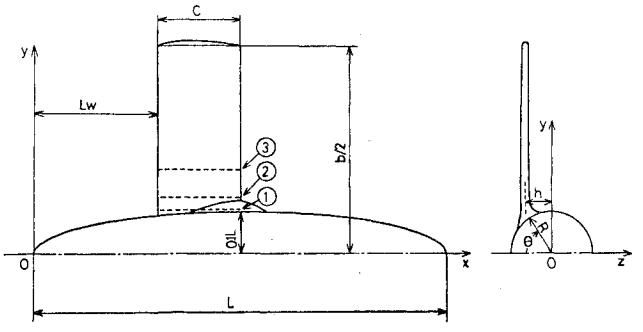

Fig. 8 Plan and front views of low-wing airplane.

experiment involving the use of tufts has been conducted by the National Aerospace Laboratory of Japan to investigate the flow characteristics over the body surface. The calculated flow velocity vectors have been found to be in good qualitative agreement with the tuft experiment, except for a small region at the tail part of the body due to the flow separation, and this seems to support our assumption about the location of C_w in the rear part of the body.

Figure 7 shows the calculated circulation distribution along the span. The abscissa is the nondimensionalized span coordinate $y/(b/2)$ and the ordinate is the nondimensionalized circulation $2\Gamma(y)/cU_\infty$ for $\alpha = 3.81$ deg, where $\Gamma(y)$ is the circulation at the spanwise station y and is obtainable by the use of Eq. (5) as follows:

$$\Gamma(y) = \varphi_+(y) - \varphi_-(y) \tag{13}$$

In this figure, the solid and dotted curves indicate the circulation distribution over the wing-body combination and the wing alone, respectively. This figure indicates that the lift has increased by the presence of the body and that the lift acting on the body is of the same order of magnitude as that acting on the corresponding part of the wing alone. It seems that the present assumption about the location of C_w over the rear part of the body should be improved from the viewpoint of the circulation distribution.

It is finally noted that the computation has been carried out by HITAC 8700/8800 of the University of Tokyo, and computing time for obtaining Figs. 5 and 6 was 190 s.

Low-Wing Airplane

Next, we proceed to investigate the flowfield around a low-wing airplane as shown in Fig. 8. The wing is of rectangular planform having aspect ratio of 5, NACA 65A010 section, no twist, and no dihedral, while the body is an ellipsoid of revolution having fineness ratio of 5. The span of the wing is taken to be equal to the length of the body. The wing is fixed to the body with zero incidence angle at $L_w = 0.3L$ and $h/R = 2/3$ as shown in Fig. 8, where h is the distance between the x axis and the middle plane of the wing, while R is the

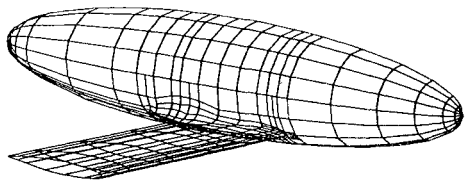

Fig. 9 Mesh division of low-wing airplane.

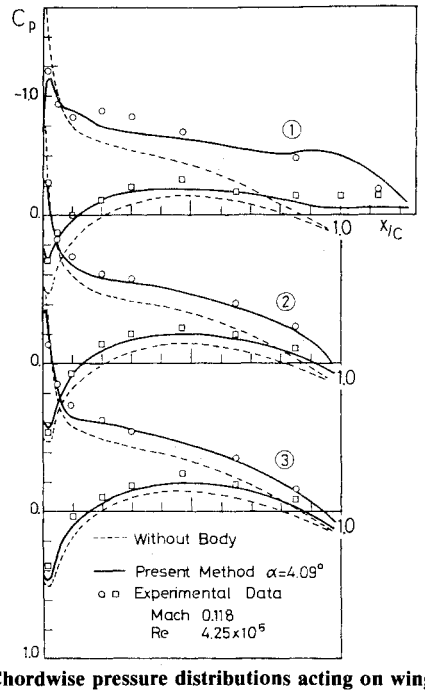

Fig. 10 Chordwise pressure distributions acting on wing at $\alpha = 4.09$ deg.

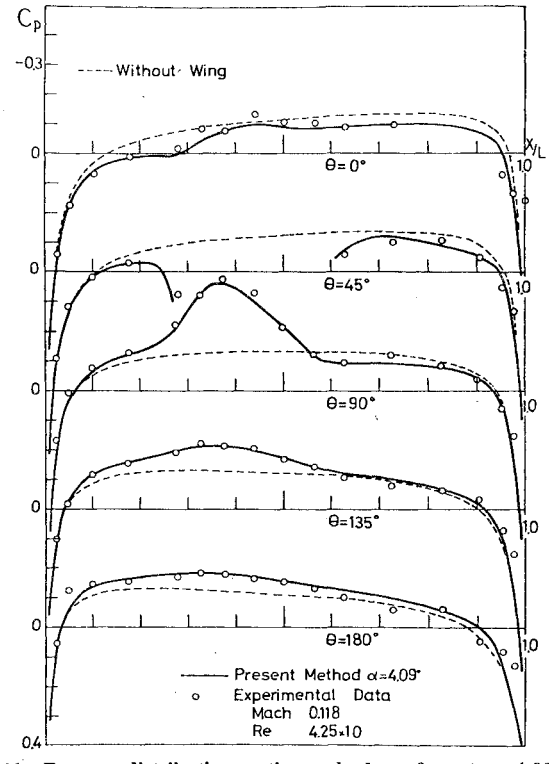

Fig. 11 Pressure distributions acting on body surface at $\alpha = 4.09$ deg.

maximum radius of the body. A fillet is attached on both sides of the wing between the wing and the body to avoid premature separation due to the interference between the wing and the body. Three spanwise stations, $y/c = 0.513$, 0.688, and 1.025, denoted by ①, ②, and ③, respectively, are shown in the same figure for later reference.

For the analysis of the low-wing airplane, we shall assume the location of C_w over the rear part of the body in such a way that it coincides with the meridian curves which are continuous with the trailing edges of the fillets, namely $\theta = \pm\cos^{-1}(h/R)$ meridians. As will be explained later, this assumption has been found to provide results which are in good agreement with our experiments. The wake surface S_w is assumed to consist of a family of semi-infinite straight lines which start from C_w and extend to infinity in the direction parallel to the uniform velocity U_∞.

Mesh division is as shown in Fig. 9. Due to the symmetry property of the present problem, only the starboard half was treated: the surface of the airplane S_A and the wake surface S_w are divided into 373 quadrilateral elements and 15 strip elements, respectively.

Figure 10 shows the pressure distributions over the wing surface at three spanwise stations, $y/c = 0.513$, 0.688, and 1.025 for $\alpha = 4.09$ deg. In the figure, the abscissa is the nondimensionalized chordwise coordinate, while the ordinate is the nondimensionalized pressure. Figure 11 shows the pressure distributions over the body surface along five meridian curves, $\theta = 0$, 45, 90, 135, and 180 deg for $\alpha = 4.09$ deg. In the figure, the abscissa is the nondimensionalized longitudinal coordinate, while the ordinate is the nondimensionalized pressure. In these figures, the solid lines represent results obtained by the present method, while discrete symbols, ○ and □, respresent experimental results obtained by the National Aerospace Laboratory of Japan.[15] It is noted that the dotted lines in Fig. 10 indicate the pressure distributions over the wing without the body, namely the wing alone, while those in Fig. 11 indicate the pressure distributions over the body without the wing, namely the body alone.

It is seen in Figs. 10 and 11 that agreement between numerical and experimental results is encouraging, and that the effect of interference among the wing, the body, and the fillet have been clearly indicated. It is added that numerical analysis has been carried out for several angles of attack and the pressure distributions obtained by the present numerical method have been found to be in good agreement with those obtained experimentally for the range of angle of attack, -4 deg $\leqq \alpha \leqq +8$ deg.

The flow velocity vectors over the body surface were also calculated, and compared with a wind tunnel experiment in which white paste was painted on the body surface to investigate the flow directions over it.[15,16] It has been found that the calculated flow velocity vectors are in good qualitative agreement with the experiments, except for a small region at the tail part of the body due to the flow separation, and this seems to support our assumption on the location of C_w in the rear part of the body. Needless to say, however, further study should be carried out to find a better and more reasonable mathematical modelling of the flowfield around the body and its wake to broaden the applicability of the present method.

It is noted that the computation was carried out by HITAC 8700/8800 of the University of Tokyo and computing time for obtaining Figs. 10 and 11 was about 200 s.

It is finally noted that the present method has been applied to several other examples, including a T-wing, an inverted T-wing, a rectangular wing with two rectangular end plates, and a cruciform wing with and without a fillet.[16-19] The agreement between numerical and experimental results have been found to be very good. These seem to have enhanced the practical value of the present finite element technique.

Conclusion

It has been shown that the finite element technique presented in the present paper provides an effective tool for numerical analysis of steady incompressible flow about a three-dimensional streamlined body such as an airplane. Numerical computations have been carried out and results have been obtained with good accuracy and in reasonable computing time, suggesting that the present technique can be used for practical design purposes.

As mentioned in the Introduction, the present paper put main emphasis on the study of the interference effect between the wing and the body. However, the present technique would provide a practical numerical method for obtaining aerodynamic characteristics such as aerodynamic derivatives of the airplane, if further work would be carried out to find a better and more reasonable mathematical modelling of the flowfield around the airplane.

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